Oliver H. Arnold, while the funds for equipment have been subscribed.

Professor Thomas S. Fiske has been designated as administrative head of the Columbia University department of mathematics for two years beginning July 1, in the place of Professor Cassius J. Keyser, who retires at his own request.

Mr. Morris M. Wells, of the University of Illinois, has been appointed instructor in the department of zoology in the University of Chicago.

The Benjamin Peirce instructorships in mathematics at Harvard University, the terms of whose establishment were recently announced in Science, have now been filled for the year 1915–16 by the appointment of Dr. Edward Kircher and Dr. George A. Pfeiffer.

DISCUSSION AND CORRESPONDENCE

THE FUNDAMENTAL EQUATION OF MECHANICS

To the Editor of Science: Professor Huntington's letter in Science of February 5 is an important contribution to the subject of the teaching of elementary dynamics, but the fact that he and Professor Hoskins are not in agreement on "the question whether $F=ma$ or $F/F'=A/A'$ is the better form in which to introduce the fundamental equation of mechanics" shows that something remains to be said on the subject. In my opinion neither of these equations ought to be considered as fundamental, for both are derived from more elementary equations.

Professor Huntington objects to $F=ma$ for certain reasons. He might have made other objections to it: for example, the equation is not true in the ordinary English system (foot-pound-second) until it is hybridized by valuing either F or m in some other unit than pounds (poundal or gee-pound) or a in "gravitals" (instead of feet) per second per second (1 gravital $= 32.174$ feet),[1] or else the letter m is explained as not being quantity of matter in pounds, but only the quotient or ratio W/g. Neither is it true in the metric kilogram-meter-second system. (I do not think the metric people have yet tried to introduce a "kilogrammal" or a "gee-kilogram.") It is of course true in the dyne-centimeter-gramme-second system, but this system is only used in higher physical theory, and it should not be inflicted on young students. The equation $F=ma$ is, however, a handy equation to work with when it is understood that m is merely a conventional symbol for W/g.

The equation $F/F'=A/A'$ may be useful for some purposes, but I agree with Professor Hoskins in not accepting it as fundamental or as the best equation to be used as an introduction of the subject. Each of the equations being open to objection, I wish that both Professor Hoskins and Professor Huntington would consider the following treatment of the subject, and let me know what objections there are to it.

Quoting Professor Huntington's words: "The first serious problem which confronts the teacher of dynamics is the problem of making the student understand the effect which a force produces when it acts on a material particle" (I would substitute the word "body" for material particle).

Let us start with the student just out of the grammar school, who has never studied physics, but who understands the simplest forms of algebraic equations, and how to make $a=F/m$ out of $F=ma$. He already knows the ordinary meaning of the words time, space, force, matter (or stuff, solid, liquid or gas). He may be told that the word "body" means a piece or chunk of stuff, and that velocity is just another name for speed. He knows that force may be measured by a spring balance, and that the quantity of matter in a body may be determined by weighing it on grocer's even-balance scale or on a platform

[1] The writer invented the "gravital" and also the "timal" ($=1/32.2$ of a second) over 20 years ago as antidotes to the "poundal," merely to serve as "horrible examples" of what might be done in the way of introducing still further confusion into our systems of units. He also invented the "massal" $= 32.2$ pounds, but that has got into some text-books disguised under the names of "gee-pound," slug, and "engineers unit of mass." The latter term is especially objectionable, for it has never been used by engineers.

scale; provided it is weighed at any place other than the imaginary "point of zero gravity."

The fundamental problem to be considered by the student is: Given a constant force F lbs. acting for T seconds on a quantity of matter W lbs., at rest at the beginning of the time, but free to move, what are the results, assuming that there is no frictional resistance?

The first result, which is already known by the boy, is motion, at a gradually increasing velocity. What the relation is between the elapsed time and the velocity may be determined by experiment. He may take a moving picture, with 50 films per second, of a body falling alongside of a rod marked with feet and inches. He may tow a boat having a load of 1,000 lbs. with a force of say 1 lb., exerted through a string and measured by a spring balance, alongside of a tow path on which a tape line is stretched; or there may be an Atwood machine in the high school on which experiments may be made. By these experiments he will learn the fundamental facts of dynamics and establish the fundamental equation. The facts are that the velocity varies directly as the time and as the force, and inversely as the quantity of matter, and the equation is $V \propto FT/W$ or $V = KFT/W$, K being a constant whose value is approximately 32, provided V is in feet per second, F and W in pounds and T in seconds.

The accurate determination of K requires the most refined experiments, involving precise measurements of both F and W, and of S, the distance traversed during the time T, from which V is determined, and precautions to eliminate resistance due to friction of air or water or of the machine used in the experiments. When these refined experiments have been made it has been found that the value of K is 32.1740, and this figure is twice the number of feet that the body would fall *in vacuo* in one second at or near latitude 45° at the sea level. It is commonly represented by g, or by g_0, to distinguish it from other values of g that may be obtained by experiments on falling bodies (or on pendulums) at other latitudes and elevations. The fundamental equation then is

$$V = FTg/W \qquad (1)$$

The velocity V is a derived quantity, derived from measurements of space (or distance) and time. If a body is moving at a uniform speed, such as the minute hand of a watch, V is a constant, and the distance varies directly as the time, and is the product of the velocity and the time, $S = VT$. But if the velocity varies directly as the time (uniformly accelerated motion), as in the case of the problem we are considering, then the distance is the product of the mean velocity and the time. Since in our problem the body starts from rest when the velocity is 0, and the velocity is V at the end of the time T, the mean velocity is $\frac{1}{2}V$ and the distance is $\frac{1}{2}VT$, whence $V = 2S/T$ and $T = 2S/V$.

The velocity V in feet per second, at the end of the time T is numerically equal to the number of feet the body would travel in one second after the expiration of the time T if the force had then ceased to act and the body continued to move at a uniform velocity.

The fundamental equation might be written $2S/T = FTg/W$, which is equivalent to $S = FT^2g/2W$, but as this is somewhat more cumbersome than the simpler-looking equation $V = FTg/W$, this latter equation is more convenient as the fundamental equation. It expresses the facts that the velocity varies directly as F and T and inversely as W, and that the velocity equals the product of F, T and g divided by W. Let us further consider the two equations $V = FTg/W$ (1) and $S = FTg/2W$ (2).

We have dealt with four elementary quantities F, T, S, W, one derived quantity V, and one constant figure 32.1740. It is understood that F is measured in standard pounds of force, the standard pound of force being the force that gravity exerts on a pound of matter at the standard location where $g = 32.1740$.

Each equation contains four variables V, F, T, W, or S, F, T, W, and in either equation if values be given to any three out of the four the fourth may be found. By ordinary algebraic transposition, or by giving new symbols to the product or quotient of two of the variables, many different equations may be derived from them, some of which are more curious than

useful. It is well not to give the student too many of them or he will become confused.

Here are some conclusions that may be derived from the equations, (1) and (2).

From (1), let $F=W$, the case of a body falling at latitude 45° at the sea level; then $V=gT$. If T also $=1$, then $V=g$, that is the velocity at the end of 1 second is g.

In the equation $V=gT$ substitute for T its value $2S/V$ and we have $V=2gS/V$, whence $V^2=2gS$. In the case of falling bodies, the height of fall H is usually substituted for S, and we obtain $V=\sqrt{2gH}$ (3).

Equation (2) with $F=W$ gives $V=\frac{1}{2}gT^2$.

From (1), by transposition we may obtain $FT=W\times V/g$ (4). The product FT is sometimes called impulse, and to the expression $W\times V/g$ is given the term momentum. It is usually written W/gV, but there is no reason why, except that it is customary, and it has been found convenient to use the letter M instead of W/g, so that the equation becomes

$$FT = MV \tag{5}$$

Impulse = Momentum

In (4) we may substitute for T its value in terms of S and V above given, viz., $T=2S/V$ and obtain $F2S/V=MV$; whence $FS=\frac{1}{2}MV^2$ (6). The product FS is called work, and the expression $\frac{1}{2}MV^2$ kinetic energy, whence work expended = kinetic energy.

Acceleration.—The quotient V/T is called the acceleration. It may be defined at the rate of increase of velocity, the word rate, unless otherwise stated, always meaning the rate with respect to time, or "time-rate." In the problem under consideration, the action of a force in a body free to move, with no retardation by friction, the acceleration is a constant, $V/T=A$. The quantity g is commonly called the acceleration due to gravity, but it also may be considered either as an abstract figure, the constant g in equation (1), or as the velocity acquired at the end of 1 second by a falling body, or as the distance a body would travel in 1 second at that same velocity if the force ceased to act and the velocity remained constant.

Equation (6) then may be written

$$F=MA \tag{7}$$

Force = M times the acceleration.

If a given particle [body] is acted on at two different times by two forces F and F', and if A and A' are the corresponding accelerations, then $\dfrac{F=MA}{F'=MA'}$ whence $F/F'=A/A'$. (8)

Equation (7) is called the fundamental equation by Professor Hoskins, while equation (8) is called fundamental by Professor Huntington, but it is shown above that they are derived from the more fundamental equation $V=FTg/W$.

Summary.—Take equation (1), $V=FTg/W$ (1). Substitute $2S/T$ for V, $S=FT^2g/2W$ (2).

Take $F=W$, then $S=\frac{1}{2}gT^2$,

and $V=\sqrt{2gH}$ (3)

From (1) by transposition $FT=WV/g$ (4)

Substitute M for W/g, $FT=MV$ (5).

In (5) substitute $2S/V$ for T,

$$FS=\tfrac{1}{2}MV^2 \tag{6}$$

In (5) substitute A for V/T, $F=MA$ (7)

Apply (7) to the case of two forces acting at different times on the same body

$$F/F'=A/A' \tag{8}$$

In this treatment the ambiguous words "weight" and "mass" have purposely been omitted.

If there is any easier way of "making the student understand the effect which a force produces when it acts on a material particle" than to have him study the above discussion and solve examples by its aid, it is very important that it should be found and incorporated in the text-books.

Wm. Kent

A COURSE IN AGRICULTURE FOR NON-TECHNICAL COLLEGES

That there is an interest in agriculture as a subject of study in colleges or higher institutions in addition to that met by the state agricultural colleges, is manifested by the introduction a few years ago into the curriculum, in certain institutions (*e. g.*, Syracuse and Miami Universities) of several subjects associated with the work of the land-grant colleges. Further evidence is shown in the